Mr. Wright's Math Extravaganza

Precalculus
Exponential and Logarithmic Functions

Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: level 3.0 and success on applications

Score	I Can Statements
4.0	☐ I can demonstrate in-depth inferences and applications that go beyond what was taught.
3.5	In addition to score 3.0 performance, partial success at score 4.0 content.
3.0	☐ I can solve exponential functions. ☐ I can solve logarithmic functions.
2.5	No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content.
2.0	☐ I can evaluate and graph exponential functions. ☐ I can evaluate and graph logarithmic functions. ☐ I can expand and condense logarithmic expressions. ☐ I can evaluate logarithmic expression using the change-of-base formula. ☐ I can solve problems using models containing exponential and logarithmic expressions.
1.5	Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content.
1.0	With help, partial success at score 2.0 content and score 3.0 content.
0.5	With help, partial success at score 2.0 content but not at score 3.0 content.
0.0	Even with help, no success.

Richard Wright

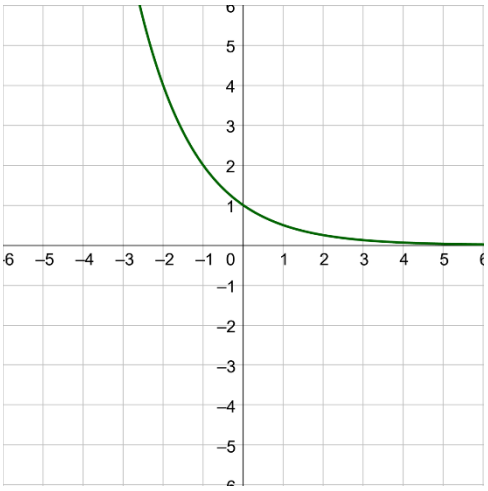
Precalculus

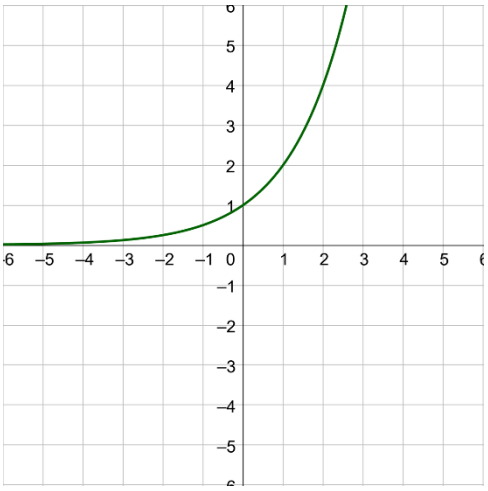
3-01 Exponential Functions

Exponential function

$$y = a \cdot b^x$$

- a is _____ amount (y-int)
- b is _____
- x is _____
- If $b > 1$
  - Exponential _____
- If $0 < b < 1$
  - Exponential _____
- Domain: _____
- Range: _____
- Horizontal Asymptote: _____
- y-intercept: _____





Transformations

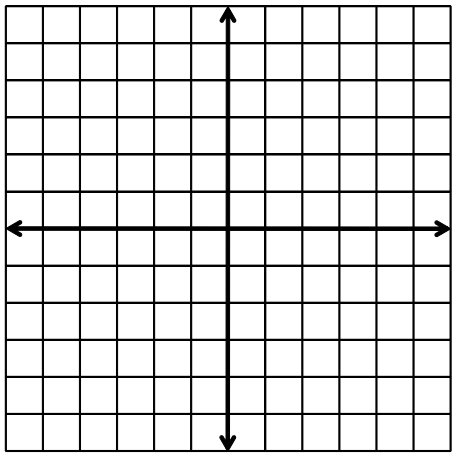
$$y = a \cdot b^{x-h} + k$$

- a _____ stretch
  - If a is negative, then _____ over x-axis
- h moves _____
- k moves _____

- Domain: _____
- Range:
  - _____ if $a > 0$
  - _____ if $a < 0$
- Horizontal Asymptote: _____
- y-int: _____ if $h = 0$

Graph by making a table

Graph $y = 4^{-x} + 3$



Exponential functions are _____

Each *x* gives a _____ *y*

Solve $16 = 2^{x+2}$

Solve $\left(\frac{1}{3}\right)^{x} = 81$

Natural Base

- $e = \left(1 + \frac{1}{n}\right)^{n}$ when $n \to \infty$
- $e \approx$ _____ ...

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = current amount
P = principle (initial amount)
r = yearly interest rate (APR)
n = number of compoundings per year
t = years

Compounded Continuously

$$A = Pe^{rt}$$

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3-02 Logarithmic Functions

$$f(x) = \log_b x$$

- "log base b of x"
- Logarithms are _____of _____functions

$$y = \log_b x \leftrightarrow x = b^y$$

- Logarithms are _____!

Evaluate

- Think "What _____of the base gives the big number?"

$\log_5 125$ $\log_2 \frac{1}{64}$

Calculator

- LOG → _____ → log
- LN → _____ → ln

Use your calculator to evaluate $\log 300$

Properties of Logarithms

- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b b^x = x$
- If $\log_b x = \log_b y$, then $x = y$

Simplify $\log_5 1$ $\log_e e$

$8^{\log_8 30}$

Solve $\log_3(x^2 + 4) = \log_3 29$

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3-03 Properties of Logarithms

Properties of Logarithms

- **Product Property:** $\log_b(uv) = \log_b u + \log_b v$
- **Quotient Property:** $\log_b\left(\frac{u}{v}\right) = \log_b u - \log_b v$
- **Power Property:** $\log_b u^n = n \log_b u$

Write each log in terms of ln 2 and ln 5.

ln 10

$\ln \frac{5}{32}$

Expand

$\log 3x^2y$

$\ln \frac{\sqrt{4x+1}}{8}$

Condense

$\frac{1}{3}\log x + 5\log(x-3)$

$4\ln(x-4) - 2\ln x$

$\frac{1}{5}(\log_3 x + \log_3(x-2))$

Change-of-Base Formula

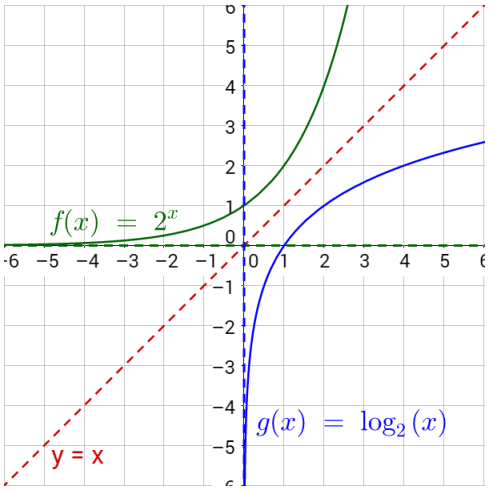
$$\log_b c = \frac{\log_a c}{\log_a b}$$

Evaluate $\log_3 17$

Precalculus 3-03 Name: _____

Graphing Logarithms

- Because logs are _____of exponentials, the *x* and *y* are _____and the graph is _____over the line *y* = *x*.



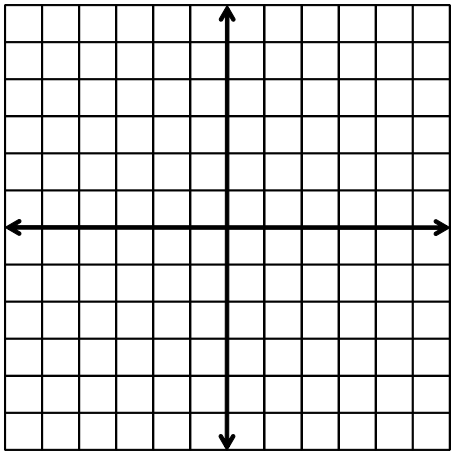
$y = \log_b(x - h)$

- Domain: _____
- Range: _____
- VA: _____
- *x*-int: _____

To graph a logarithm

1. Find and graph the vertical _____
2. Make a _____
3. Use _____ formula
   $\log_b x = \frac{\log x}{\log b}$
4. Or use the _____function on some TI graphing calcs
   MATH → logBASE

Graph $y = \log_2(x + 1)$



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3-04 Solving Exponential and Logarithmic Equations

Solve Exponential Equations

Shortcut Method
1-to-1 method (rewrite with the same base)

$\left(\frac{1}{5}\right)^{x} = 125$

General Method
_____of both sides

$5 - 3e^{x} = 2$

$6(2^{t+5}) + 4 = 11$

$e^{2x} - 7e^{x} + 12 = 0$

Logarithmic Equations

Shortcut Method
1-to-1 Property

$\ln x - \ln 3 = 0$

General Method

_____ both sides

$6 + 3\ln x = 4$

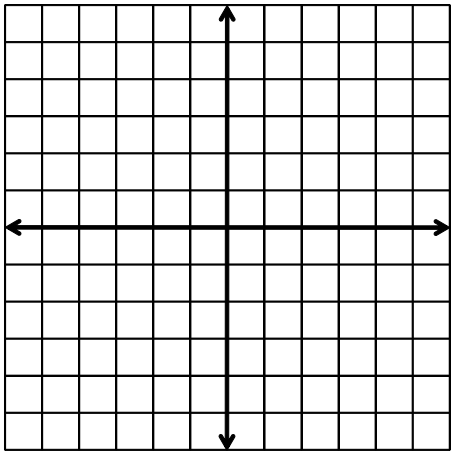
$\log_4 x + \log_4(x - 9) = 1$

Graphical method

If the other methods don't apply

- Make _____
- Find the _____

Solve $\log_2 x = \ln 2x$



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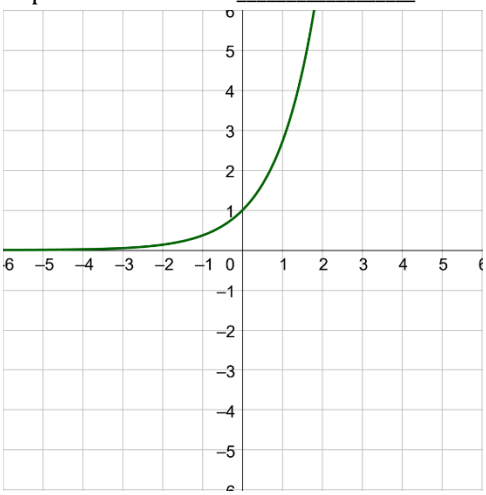
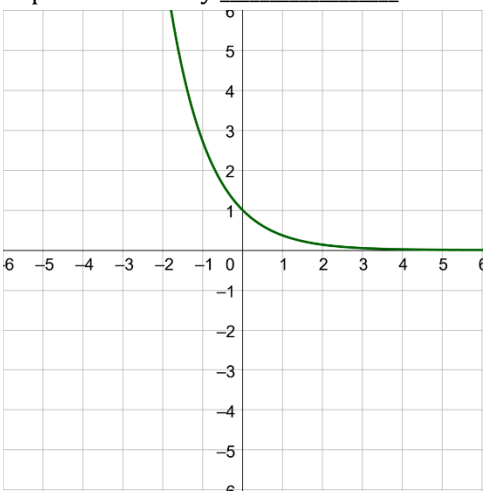
3-05 Exponential and Logarithmic Models

Exponential Growth and Decay

Exponential Growth _____



Exponential Decay _____



Suppose a population growing according to the model $P = 800e^{0.03t}$ where t is in years.
What is the initial size?

How long to double?

Radioactive decay

- $y = ae^{-bx}$
- $A = A_0e^{kt}$

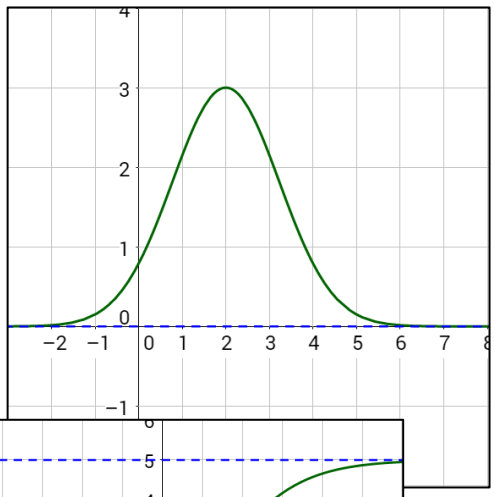
Half-life

- Time it takes for _____ of the material to _____

C^{14} has a half-life of 5700 years. If a sample starts with 3 g of C^{14}, how much will remain after 100 years?

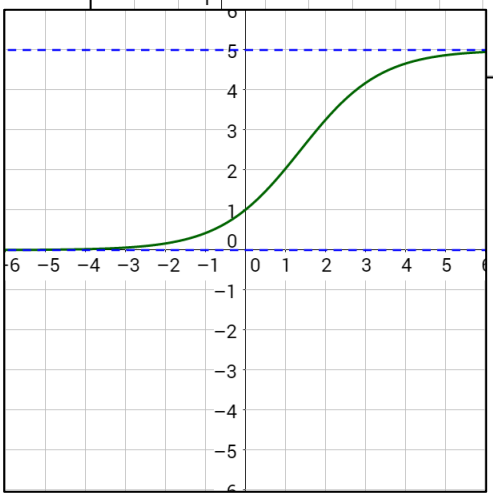
Gaussian Model "The Curve"

- _____Distribution
- $y = ae^{-\frac{(x-b)^2}{c}}$



Logistic Growth Model

- Used for _____
- $y = \frac{a}{1+be^{-rx}}$



Logarithmic Models

- $y = a + b \ln x$
- $y = a + b \log x$
- Richter Scale
  - _____magnitude
- Decibels
  - _____of sound